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NEUTROSOPHIC gsα* - SEPARATION AXIOMS IN NEUTROSOPHIC TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we newly introduced the concept based on $N_{eu}-$ separation axioms namely N_{eu}gsα* - separation axioms in $N_{eu}-$Topological Spaces . It includes the concept of N_{eu}gsα*- T_1 , N_{eu}gsα*- T_2 and N_{eu}gsα*- T_3 spaces . In additionally , we discussed about the characterizations and properties of these axioms with already existing N_{eu}- separation axioms. The main goal of this paper is to introduce a new definition N_{eu}gsα*- closed set . This definition forms a topological space . So , all the basic theorems in topological spaces are true for my definition .

Keywords: N_{eu}gsα*- OS , N_{eu}gsα*- CS , N_{eu}gsα* - T_0 space , N_{eu}gsα* - T_1 space , N_{eu}gsα* - T_2 space , N_{eu}gsα* - T_3 space , N_{eu}gsα* - regular space , N_{eu}gsα* - hausdorff space .

1. INTRODUCTION

As a generalization of fuzzy sets introduced by L.A.Zadeh's[9] and intuitionistic fuzzy set introduced by K.Atanassov's[4] . Then the idea of $N_{eu}-$ set theory was introduced by F.Smarandache[6] . It consists of three components namely truth , indeterminancy and false membership function . Ahu Acikgoz and Ferhat Esenbel[1] has newly introduced the concept of $N_{eu}-$ separation axioms in $N_{eu}-$ topological space. The real life application of $N_{eu}-$ topology is applied in Information Systems , Applied Mathematics etc .

In this paper, we introduce some new concepts in $N_{eu}-$ topological spaces such as N_{eu}gsα* - T_i (i=0,...,3) spaces , N_{eu}gsα* - regular space , N_{eu}gsα*- hausdorff space . The goal of this paper is to achieve the separation axiom of my definition is the best one when compared with the $N_{eu}-$ topological spaces . Every $N_{eu}-$ separation axioms will be the weaker set for my N_{eu}gsα* - separation axioms . The scientific contribution of my paper is , it is applicable to all theorems in neutrosophic topological space .

2. Preliminaries

Definition 2.1:[8] Let $\mathbb{P}$ be a non-empty fixed set . A $N_{eu}-$ set Ƕ on the universe $\mathbb{P}$ is defined as Ƕ $=\left\{\langle p, \left(t_{Ƕ}(p), i_{Ƕ}(p), f_{Ƕ}(p)\right)\rangle : p \in P\right\}$ where $t_{Ƕ}(p)$, $i_{Ƕ}(p)$, $f_{Ƕ}(p)$ represent the degree of membership function , the degree of indeterminacy and the degree of non-membership function respectively for each element $p \in P$ to the set Ƕ . Also , $t_{Ƕ}$, $i_{Ƕ}$, $f_{Ƕ}$: $\mathbb{P} \rightarrow\ ]^-0 , 1^+[$ and $^-0 \leq t_{Ƕ}(p) + i_{Ƕ}(p) + f_{Ƕ}(p) \leq 3^+$. Set of all $N_{eu}-$ set over $\mathbb{P}$ is denoted by $N_{eu}(\mathbb{P})$.

Definition 2.2:[8] Let $\mathbb{P}$ be a non-empty set. Ⱥ $=\left\{\langle p, \left(t_{Ⱥ}(p), i_{Ⱥ}(p), f_{Ⱥ}(p)\right)\rangle : p \in P\right\}$ and

Ƀ $=\left\{\langle p, \left(t_{Ƀ}(p), i_{Ƀ}(p), f_{Ƀ}(p)\right)\rangle : p \in P\right\}$ are $N_{eu}-$ sets , then

(i) $A \subseteq B$ if $t_A(p) \leq t_B(p),\ i_A(p) \leq i_B(p), f_A(p) \geq f_B(p)$ for all $p \in P$.

(ii) $A \cap B = \{\langle p, ((t_A(p), t_B(p)), (i_A(p), i_B(p)), (f_A(p), f_B(p)))\rangle : p \in P\}$.

(iii) $A \cup B = \{\langle p, ((t_A(p), t_B(p)) (i_A(p), i_B(p)), (f_A(p), f_B(p)))\rangle : p \in P\}$.

(iv) $A^c = \{\langle p, (f_A(p), 1 - i_A(p), t_A(p))\rangle : p \in P\}$.

(v) $0_{N_{eu}} = \{\langle p, (0, 0, 1)\rangle : p \in P\}$ and $1_{N_{eu}} = \{\langle p, (1, 1, 0)\rangle : p \in P\}$.

Definition 2.3:[8] A N_{eu} − topology ($N_{eu}T$) on a non-empty set $\mathbb{P}$ is a family $\tau_{N_{eu}}$ of neutrosophic sets in $\mathbb{P}$ satisfying the following axioms ,

(i) $0_{N_{eu}}, 1_{N_{eu}} \in \tau_{N_{eu}}$.

(ii) $A_1 \cap A_2 \in \tau_{N_{eu}}$ for any $A_1, A_2 \in \tau_{N_{eu}}$.

(iii) $\cup A_i \in \tau_{N_{eu}}$ for every family $\{A_i / i \in \Omega\} \subseteq \tau_{N_{eu}}$.

In this case , the ordered pair $(P, \tau_{N_{eu}})$ or simply $\mathbb{P}$ is called a N_{eu} − topological space ($N_{eu}TS$) . The elements of $\tau_{N_{eu}}$ is neutrosophic open set ($N_{eu} - OS$) and $\tau_{N_{eu}}^c$ is neutrosophic closed set ($N_{eu} - CS$) .

Definition 2.4:[2] A neutrosophic set A in a N_{eu}TS $(P, \tau_{N_{eu}})$ is called a neutrosophic generalized semi alpha star closed set ($N_{eu}gs\alpha^* - CS$) if $N_{eu}\alpha - int(N_{eu}\alpha - cl(A)) \subseteq N_{eu} - int(G)$, whenever $A \subseteq G$ and G is $N_{eu}\alpha^*$ − open set .

Definition 2.5:[3] A $N_{eu}TS$ $(P, \tau_{N_{eu}})$ is called a $N_{eu}gs\alpha^* - T_{\frac{1}{2}}$ space if every $N_{eu}gs\alpha^* - CS$ in $(P, \tau_{N_{eu}})$ is a $N_{eu} - CS$ in $(P, \tau_{N_{eu}})$.

Definition 2.6:[5] Let Q be a nonempty set . If t, i, f are real standard or nonstandard subsets of $]^-0, 1^+[$, then the N_{eu} − set $q_{t,i,f}$ given by

$$q_{t,i,f}(q_p) = \{(t, i, f),\ if\ q = q_p\ (0, 0, 1),\ if\ q \neq q_p$$

is called a neutrosophic point ($N_{eu} - pts$) in Q and $q_p \in Q$ is called the support of $q_{t,i,f}$. Here t denotes the degree of membership value , i denotes the degree of indeterminacy and f denotes the degree of non-membership value of $q_{t,i,f}$.

Definition 2.7: A $N_{eu}TS$ $(P, \tau_{N_{eu}})$ is named as

(1) $N_{eu} - T_0$ space[1] if for each pair of distinct $N_{eu} - pts\ p_{t,i,f}$ and $q_{t,i,f}$ in P , then there exists $N_{eu} - OS$ E in $(P, \tau_{N_{eu}})$ such that $p_{t,i,f} \in E$, $q_{t,i,f} \notin E$ (or) $p_{t,i,f} \notin E$, $q_{t,i,f} \in E$.

(2) $N_{eu}\alpha - T_0$ space[5] if for each pair of distinct $N_{eu} - pts$ $p_{t,i,f}$ and $q_{t,i,f}$ in P, then there exists $N_{eu}\alpha - OS$ $\mathbb{E}$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in \mathbb{E}$, $q_{t,i,f} \notin \mathbb{E}$ (or) $p_{t,i,f} \notin \mathbb{E}$, $q_{t,i,f} \in \mathbb{E}$.

(3) $N_{eu} - T_1$ space[1] if for each pair of distinct $N_{eu} - pts$ $p_{t,i,f}$ and $q_{t,i,f}$ in P, then there exists $N_{eu} - OS$ $\mathbb{E}$ and $\mathbb{C}$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in \mathbb{E}$, $q_{t,i,f} \notin \mathbb{E}$ and $p_{t,i,f} \notin \mathbb{C}$, $q_{t,i,f} \in \mathbb{C}$.

(4) $N_{eu}\alpha - T_1$ space[5] if for each pair of distinct $N_{eu} - pts$ $p_{t,i,f}$ and $q_{t,i,f}$ in P, then there exists $N_{eu}\alpha - OS$ $\mathbb{E}$ and $\mathbb{C}$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in \mathbb{E}$, $q_{t,i,f} \notin \mathbb{E}$ and $p_{t,i,f} \notin \mathbb{C}$, $q_{t,i,f} \in \mathbb{C}$.

(5) $N_{eu} - T_2$ space[1] if for each pair of distinct $N_{eu} - pts$ $p_{t,i,f}$ and $q_{t,i,f}$ in P, then there exists $N_{eu} - OS$ $\mathbb{E}$ and $\mathbb{C}$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in \mathbb{E}$, $q_{t,i,f} \in \mathbb{C}$ and $\mathbb{E} \cap \mathbb{C} = 0_{N_{eu}}$.

(6) $N_{eu}\alpha - T_2$ space[5] if for each pair of distinct $N_{eu} - pts$ $p_{t,i,f}$ and $q_{t,i,f}$ in P, then there exists $N_{eu}\alpha - OS$ $\mathbb{E}$ and $\mathbb{C}$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in \mathbb{E}$, $q_{t,i,f} \in \mathbb{C}$ and $\mathbb{E} \cap \mathbb{C} = 0_{N_{eu}}$.

(7) N_{eu} − regular space[1] if for any $N_{eu} - pts$ $p_{t,i,f}$ in P and any $N_{eu} - CS$ $\mathbb{E}$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \notin \mathbb{E}$, then there exists $N_{eu} - OS$ $\mathbb{C}_1$ and $\mathbb{C}_2$ such that $p_{t,i,f} \in \mathbb{C}_1$, $\mathbb{E} \subseteq \mathbb{C}_2$ and $\mathbb{C}_1 \cap \mathbb{C}_2 = 0_{N_{eu}}$.

(8) $N_{eu} - T_3$ space[1] if it is both N_{eu} − regular and $N_{eu} - T_1$ space.

3. Neutrosophic $gs\alpha^* - T_0$ Space

Definition 3.1: A $N_{eu}TS$ $\left(P, \tau_{N_{eu}}\right)$ is said to be $N_{eu}gs\alpha^* - T_0$ space if for each pair of distinct $N_{eu} - pts$ $p_{t,i,f}$ and $q_{t,i,f}$ in P, then there exists $N_{eu}gs\alpha^* - OS$ $\mathbb{E}$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in \mathbb{E}$, $q_{t,i,f} \notin \mathbb{E}$ (or) $p_{t,i,f} \notin \mathbb{E}$, $q_{t,i,f} \in \mathbb{E}$.

Example 3.2: Let $\mathbb{P} = \{p, q\}$ and $\tau_{N_{eu}} = \left\{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\right\}$, where $\mathbb{A} = \{\langle p, (0.4, 0.6, 0.2)\rangle, \langle q, (0.3, 0.7, 0.2)\rangle\}$ is a N_{eu}TS on $\left(P, \tau_{N_{eu}}\right)$. Let $p_{0.4, 0.6, 0.2}$, $q_{0.3, 0.6, 0.2}$ be any two distinct $N_{eu} - pts$ in P. Then there exist $N_{eu}gs\alpha^* - OS$ $\mathbb{E}$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0.4, 0.6, 0.2} \in \mathbb{E}$, $q_{0.3, 0.6, 0.2} \notin \mathbb{E}$, where $\mathbb{E} = \{\langle p, (0.4, 0.6, 0.2)\rangle, \langle q, (0.3, 0.7, 0.2)\rangle\} \Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^* - T_0$ space.

Theorem 3.3: Every $N_{eu} - T_0$ space is $N_{eu}gs\alpha^* - T_0$ space, but not conversely.

Proof: Let $\left(P, \tau_{N_{eu}}\right)$ be any $N_{eu}TS$ in $\left(P, \tau_{N_{eu}}\right)$. Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu} - T_0$ space , then for each pair of distinct $N_{eu} - pts\ p_{t,i,f}$ and $q_{t,i,f}$ in $P \Rightarrow$ there exists $N_{eu} - OS\ Ɇ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in Ɇ$, $q_{t,i,f} \notin Ɇ$ (or) $p_{t,i,f} \notin Ɇ$, $q_{t,i,f} \in Ɇ \Rightarrow$ there exists $N_{eu}gs\alpha^* - OS\ Ɇ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in Ɇ$, $q_{t,i,f} \notin Ɇ$ (or) $p_{t,i,f} \notin Ɇ$, $q_{t,i,f} \in Ɇ \Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^* - T_0$ space .

Example 3.4: Let $\mathbb{P} = \{p, q\}$ and $\tau_{N_{eu}} = \left\{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ\right\}$, where $Ⱥ =$ $\{\langle p, (0.4, 0.5, 0.7)\rangle, \langle q, (0.3, 0.4, 0.8)\rangle\}$ is a $N_{eu}TS$ on $\left(P, \tau_{N_{eu}}\right)$. Let $p_{0.7, 0.5, 0.4}$, $q_{0.7, 0.8, 0.5}$ be any two distinct $N_{eu} - pts$ in P . Then there exist $N_{eu}gs\alpha^* - OS\ Ɇ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0.7, 0.5, 0.4} \in Ɇ$, $q_{0.7, 0.8, 0.5} \notin Ɇ$, where $Ɇ = \{\langle p, (0.7, 0.5, 0.4)\rangle, \langle q, (0.8, 0.6, 0.3)\rangle\} \Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^* - T_0$ space. But $\left(P, \tau_{N_{eu}}\right)$ is not $N_{eu} - T_0$ space , because there exists $N_{eu} - OS\ Ȼ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0.7, 0.5, 0.4} \notin Ȼ$, $q_{0.7, 0.8, 0.5} \notin Ȼ$, where $Ȼ = Ⱥ$.

Theorem 3.5: Let $\left(P, \tau_{N_{eu}}\right)$ be any $N_{eu}TS$. Then $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu} - T_0$ space if $\left(P, \tau_{N_{eu}}\right)$ is both $N_{eu}gs\alpha^* - T_0$ space and $N_{eu}gs\alpha^* - T_{\frac{1}{2}}$ space .

Proof: Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^* - T_0$ space , then for each pair of distinct $N_{eu} - pts\ p_{t,i,f}$ and $q_{t,i,f}$ in P. Now , there exists $N_{eu}gs\alpha^* - OS\ Ɇ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in Ɇ$, $q_{t,i,f} \notin Ɇ$ (or) $p_{t,i,f} \notin Ɇ$, $q_{t,i,f} \in Ɇ$. Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^* - T_{\frac{1}{2}}$ space , then there exists $N_{eu} - OS\ Ɇ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in Ɇ$, $q_{t,i,f} \notin Ɇ$ (or) $p_{t,i,f} \notin Ɇ$, $q_{t,i,f} \in Ɇ \Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu} - T_0$ space .

Theorem 3.6: Every $N_{eu}\alpha - T_0$ space is $N_{eu}gs\alpha^* - T_0$ space , but not conversely .

Proof: Let $\left(P, \tau_{N_{eu}}\right)$ be any $N_{eu}TS$ in $\left(P, \tau_{N_{eu}}\right)$. Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}\alpha - T_0$ space , then for each pair of distinct $N_{eu} - pts\ p_{t,i,f}$ and $q_{t,i,f}$ in P and there exists $N_{eu}\alpha - OS\ Ɇ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in Ɇ$, $q_{t,i,f} \notin Ɇ$ (or) $p_{t,i,f} \notin Ɇ$, $q_{t,i,f} \in Ɇ \Rightarrow$ there exists $N_{eu}gs\alpha^* - OS$ such that $p_{t,i,f} \in Ɇ$, $q_{t,i,f} \notin Ɇ$ (or) $p_{t,i,f} \notin Ɇ$, $q_{t,i,f} \in Ɇ \Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^* - T_0$ space .

Example 3.7: Let $\mathbb{P} = \{p, q\}$ and $\tau_{N_{eu}} = \left\{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ\right\}$, where $Ⱥ =$ $\{\langle p, (0.5, 0.3, 0.8)\rangle, \langle q, (0.6, 0.4, 0.7)\rangle\}$ is a $N_{eu}TS$ on $\left(P, \tau_{N_{eu}}\right)$. Let $p_{0.8, 0.7, 0.5}$, $q_{0.5, 0.6, 0.5}$ be any two distinct $N_{eu} - pts$ in P . Then there exist $N_{eu}gs\alpha^* - OS\ Ɇ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0.8, 0.7, 0.5} \in Ɇ$,

$q_{0.5\,,\,0.6\,,\,0.5} \notin Ɇ$, where $Ɇ = \{\langle p,\ (0.8,\ 0.7,\ 0.5)\rangle,\ \langle q\ ,\ (0.7,\ 0.6,\ 0.6)\rangle\} \Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{0}$ space . But $\left(P, \tau_{N_{eu}}\right)$ is not $N_{eu}\alpha - T_{0}$ space , because there exists $N_{eu}\alpha - OS$ $Ȼ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0.8\,,\,0.7\,,\,0.5} \notin Ȼ$, $q_{0.5\,,\,0.6\,,\,0.5} \notin Ȼ$, where $Ȼ = Ⱥ$.

Theorem 3.8: Let $\left(P, \tau_{N_{eu}}\right)$ be any $N_{eu}TS$. Then $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}\alpha - T_{0}$ space if $\left(P, \tau_{N_{eu}}\right)$ is both $N_{eu}gs\alpha^{*} - T_{0}$ space and $N_{eu}gs\alpha^{*} - T_{\frac{1}{2}}$ space .

Proof: Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{0}$ space , then for each pair of distinct $N_{eu} - pts$ $p_{t,i,f}$ and $q_{t,i,f}$ in P $\Rightarrow$ there exists $N_{eu}gs\alpha^{*} - OS$ $Ɇ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in Ɇ$, $q_{t,i,f} \notin Ɇ$ (or) $p_{t,i,f} \notin Ɇ$, $q_{t,i,f} \in Ɇ$. Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{\frac{1}{2}}$ space , then there exists $N_{eu} - OS$ $Ɇ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in Ɇ$, $q_{t,i,f} \notin Ɇ$ (or) $p_{t,i,f} \notin Ɇ$, $q_{t,i,f} \in Ɇ$ $\Rightarrow$ then there exists $N_{eu}\alpha - OS$ $Ɇ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in Ɇ$, $q_{t,i,f} \notin Ɇ$ (or) $p_{t,i,f} \notin Ɇ$, $q_{t,i,f} \in Ɇ$ $\Rightarrow$ $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}\alpha - T_{0}$ space .

Theorem 3.9: Let $\left(P, \tau_{N_{eu}}\right)$ be any $N_{eu}TS$ and $p_{t,i,f}$, $q_{t,i,f}$ be any two distinct $N_{eu} - pts$ in P . If there exists $N_{eu}gs\alpha^{*} - OS$ $Ɇ$ in $\left(P, \tau_{N_{eu}}\right)$ such that either $p_{t,i,f} \in Ɇ$, $q_{t,i,f} \in Ɇ^{c}$ (or) $p_{t,i,f} \in Ɇ^{c}$, $q_{t,i,f} \in Ɇ$, then $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{0}$ space .

Proof: Let $p_{t,i,f}$, $q_{t,i,f}$ be any two distinct $N_{eu} - pts$ in P and $Ɇ$ be $N_{eu}gs\alpha^{*} - OS$ in $\left(P, \tau_{N_{eu}}\right)$ such that either $p_{t,i,f} \in Ɇ$, $q_{t,i,f} \in Ɇ^{c}$ (or) $p_{t,i,f} \in Ɇ^{c}$, $q_{t,i,f} \in Ɇ$ $\Rightarrow p_{t,i,f} \in Ɇ$, $q_{t,i,f} \notin Ɇ$ (or) $p_{t,i,f} \notin Ɇ$, $q_{t,i,f} \in Ɇ$ (since $Ɇ \neq Ɇ^{c}$) $\Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{0}$ space .

Theorem 3.10: A $N_{eu} -$ subspace[7] $\left(Q,\ \sigma_{N_{eu}}\right)$ of a $N_{eu}gs\alpha^{*} - T_{0}$ space $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{0}$ space .

Proof: Let $p_{t,i,f}$, $q_{t,i,f}$ be any two distinct $N_{eu} - pts$ in Q . Then $p_{t,i,f}$, $q_{t,i,f}$ be any two distinct $N_{eu} - pts$ in P . Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{0}$ space , then there exists $N_{eu}gs\alpha^{*} - OS$ $Ɇ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in Ɇ$, $q_{t,i,f} \notin Ɇ$ (or) $p_{t,i,f} \notin Ɇ$, $q_{t,i,f} \in Ɇ$. Given $p_{t,i,f} \in Q \cap Ɇ = Ɇ$ and $q_{t,i,f} \notin Q \cap Ɇ = Ɇ$, where $Ɇ$ is $N_{eu}gs\alpha^{*} - OS$ in $\left(Q,\ \sigma_{N_{eu}}\right) \Rightarrow \left(Q,\ \sigma_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{0}$ space .

4. Neutrosophic $gs\alpha^{*} - T_{1}$ Space

Definition 4.1: A $N_{eu}TS$ $\left(P, \tau_{N_{eu}}\right)$ is said to be $N_{eu}gs\alpha^{*} - T_{1}$ space if for each pair of distinct $N_{eu} - pts$ $p_{t,i,f}$ and $q_{t,i,f}$ in P , then there exists $N_{eu}gs\alpha^{*} - OS$ $\mathbb{E}$ and $\mathbb{C}$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in \mathbb{E}$, $q_{t,i,f} \notin \mathbb{E}$ and $p_{t,i,f} \notin \mathbb{C}$, $q_{t,i,f} \in \mathbb{C}$.

Example 4.2: Let $\mathbb{P}$ $=\{p, q\}$ and $\tau_{N_{eu}} = \left\{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\right\}$, where $\mathbb{A}= \{\langle p, (0.6, 0.8, 0.4)\rangle, \langle q, (0.7, 0.6, 0.3)\rangle\}$ is a $N_{eu}TS$ on $\left(P, \tau_{N_{eu}}\right)$. Let $p_{0.7, 0.8, 0.2}$, $q_{0.7, 0.8, 0.3}$ be any two distinct $N_{eu} - pts$ in P . Then there exist $N_{eu}gs\alpha^{*} - OS$ $\mathbb{E}$ and $\mathbb{C}$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0.7, 0.8, 0.2} \in \mathbb{E}$, $q_{0.7, 0.8, 0.3} \notin \mathbb{E}$ and $p_{0.7, 0.8, 0.2} \notin \mathbb{C}$, $q_{0.7, 0.8, 0.3} \in \mathbb{C}$,where $\mathbb{E} = \{\langle p, (0.7, 0.8, 0.2)\rangle, \langle q, (0.8, 0.6, 0.1)\rangle\}$ and $\mathbb{C} = \{\langle p, (0.8, 0.9, 0.3)\rangle, \langle q, (0.7, 0.8, 0.3)\rangle\} \Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{1}$ space .

Theorem 4.3: Every $N_{eu} - T_{1}$ space is $N_{eu}gs\alpha^{*} - T_{1}$ space , but not conversely .

Proof: Let $\left(P, \tau_{N_{eu}}\right)$ be any $N_{eu}TS$ in $\left(P, \tau_{N_{eu}}\right)$. Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu} - T_{1}$ space , then for each pair of distinct $N_{eu} - pts$ $p_{t,i,f}$ and $q_{t,i,f}$ in P and there exists $N_{eu} - OS$ $\mathbb{E}$ and $\mathbb{C}$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in \mathbb{E}$, $q_{t,i,f} \notin \mathbb{E}$ and $p_{t,i,f} \notin \mathbb{C}$, $q_{t,i,f} \in \mathbb{C} \Rightarrow$ there exists $N_{eu}gs\alpha^{*} - OS$ $\mathbb{E}$ and $\mathbb{C}$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in \mathbb{E}$, $q_{t,i,f} \notin \mathbb{E}$ and $p_{t,i,f} \notin \mathbb{C}$, $q_{t,i,f} \in \mathbb{C} \Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{1}$ space .

Example 4.4: Let $\mathbb{P}$ $=\{p, q\}$ and $\tau_{N_{eu}} = \left\{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\right\}$, where $\mathbb{A}= \{\langle p, (0.3, 0.2, 0.8)\rangle, \langle q, (0.4, 0.4, 0.6)\rangle\}$ is a $N_{eu}TS$ on $\left(P, \tau_{N_{eu}}\right)$. Let $p_{0.3, 0.2, 0.8}$, $q_{0.5, 0.7, 0.2}$ be any two distinct $N_{eu} - pts$ in P . Then there exist $N_{eu}gs\alpha^{*} - OS$ $\mathbb{E}$ and $\mathbb{C}$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0.3, 0.2, 0.8} \in \mathbb{E}$, $q_{0.5, 0.7, 0.2} \notin \mathbb{E}$ and $p_{0.3, 0.2, 0.8} \notin \mathbb{C}$, $q_{0.5, 0.7, 0.2} \in \mathbb{C}$, where $\mathbb{E} = \mathbb{A}$ and $\mathbb{C} = \{\langle p, (0.5, 0.3, 0.6)\rangle, \langle q, (0.5\ 0.7, 0.2)\rangle\} \Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{1}$ space . But $\left(P, \tau_{N_{eu}}\right)$ is not $N_{eu} - T_{1}$ space , because there exists $N_{eu} - OS$ $\mathbb{E}_{1}$ and $\mathbb{C}_{1}$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0.3, 0.2, 0.8} \in \mathbb{E}_{1}$, $q_{0.5, 0.7, 0.2} \notin \mathbb{E}_{1}$ and $p_{0.3, 0.2, 0.8} \notin \mathbb{C}_{1}$, $q_{0.5, 0.7, 0.2} \notin \mathbb{C}_{1}$, where $\mathbb{E}_{1} = \mathbb{A}$ and $\mathbb{C}_{1} = 1_{N_{eu}}$.

Theorem 4.5: Let $\left(P, \tau_{N_{eu}}\right)$ be any $N_{eu}TS$. Then $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu} - T_{1}$ space if $\left(P, \tau_{N_{eu}}\right)$ is both $N_{eu}gs\alpha^{*} - T_{1}$ space and $N_{eu}gs\alpha^{*} - T_{\frac{1}{2}}$ space .

Proof: Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{1}$ space , then for each pair of distinct $N_{eu} - pts$ $p_{t,i,f}$ and $q_{t,i,f}$ in P $\Rightarrow$ there exists $N_{eu}gs\alpha^{*} - OS$ $\mathbb{E}$ and $\mathbb{C}$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in \mathbb{E}$, $q_{t,i,f} \notin \mathbb{E}$ and $p_{t,i,f} \notin \mathbb{C}$, $q_{t,i,f} \in \mathbb{C}$

. Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{\frac{1}{2}}$ space , then there exists $N_{eu} - OS$ $Ɇ$ and $Ȼ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in Ɇ$, $q_{t,i,f} \notin Ɇ$ and $p_{t,i,f} \notin Ȼ$, $q_{t,i,f} \in Ȼ \Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu} - T_1$ space .

Theorem 4.6: Every $N_{eu}\alpha - T_1$ space is $N_{eu}gs\alpha^{*} - T_1$ space , but not conversely .

Proof: Let $\left(P, \tau_{N_{eu}}\right)$ be any $N_{eu}TS$ in $\left(P, \tau_{N_{eu}}\right)$. Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}\alpha - T_1$ space , then for each pair of distinct $N_{eu} - pts$ $p_{t,i,f}$ and $q_{t,i,f}$ in $P \Rightarrow$ there exists $N_{eu}\alpha - OS$ $Ɇ$ and $Ȼ$ such that $p_{t,i,f} \in Ɇ$, $q_{t,i,f} \notin Ɇ$ and $p_{t,i,f} \notin Ȼ$, $q_{t,i,f} \in Ȼ \Rightarrow$ there exists $N_{eu}gs\alpha^{*} - OS$ $Ɇ$ and $Ȼ$ such that $p_{t,i,f} \in Ɇ$, $q_{t,i,f} \notin Ɇ$ and $p_{t,i,f} \notin Ȼ$, $q_{t,i,f} \in Ȼ \Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_1$ space .

Example 4.7: Let $\mathbb{P} = \{p, q\}$ and $\tau_{N_{eu}} = \left\{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ\right\}$, where $Ⱥ = \{\langle p, (0.2, 0.4, 0.6)\rangle, \langle q, (0.4, 0.5, 0.7)\rangle\}$ is a $N_{eu}TS$ on $\left(P, \tau_{N_{eu}}\right)$. Let $p_{0.2,0.4,0.6}$, $q_{0.6,0.4,0.2}$ be any two distinct $N_{eu} - pts$ in P . Then there exist $N_{eu}gs\alpha^{*} - OS$ $Ɇ$ and $Ȼ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0.2,0.4,0.6} \in Ɇ$, $q_{0.6,0.4,0.2} \notin Ɇ$ and $p_{0.2,0.4,0.6} \notin Ȼ$, $q_{0.6,0.4,0.2} \in Ȼ$, where $Ɇ = Ⱥ$ and $Ȼ = \{\langle p, (0.2, 0.3, 0.7)\rangle, \langle q, (0.6, 0.4, 0.2)\rangle\} \Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_1$ space . But $\left(P, \tau_{N_{eu}}\right)$ is not $N_{eu}\alpha - T_1$ space , because there exists $N_{eu}\alpha - OS$ $Ɇ_1$ and $Ȼ_1$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0.2,0.4,0.6} \in Ɇ_1$, $q_{0.6,0.4,0.2} \notin Ɇ_1$ and $p_{0.2,0.4,0.6} \notin Ȼ_1$, $q_{0.6,0.4,0.2} \notin Ȼ_1$, where $Ɇ_1 = Ⱥ$ and $Ȼ_1 = 1_{N_{eu}}$.

Theorem 4.8: Let $\left(P, \tau_{N_{eu}}\right)$ be any $N_{eu}TS$. Then $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}\alpha - T_1$ space if $\left(P, \tau_{N_{eu}}\right)$ is both $N_{eu}gs\alpha^{*} - T_1$ space and $N_{eu}gs\alpha^{*} - T_{\frac{1}{2}}$ space .

Proof: Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_1$ space , then for each pair of distinct $N_{eu} - pts$ $p_{t,i,f}$ and $q_{t,i,f}$ in P $\Rightarrow$ there exists $N_{eu}gs\alpha^{*} - OS$ $Ɇ$ and $Ȼ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in Ɇ$, $q_{t,i,f} \notin Ɇ$ and $p_{t,i,f} \notin Ȼ$, $q_{t,i,f} \in Ȼ$. Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{\frac{1}{2}}$ space , then there exists $N_{eu} - OS$ $Ɇ$ and $Ȼ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in Ɇ$, $q_{t,i,f} \notin Ɇ$ and $p_{t,i,f} \notin Ȼ$, $q_{t,i,f} \in Ȼ \Rightarrow$ then there exists $N_{eu}\alpha - OS$ $Ɇ$ and $Ȼ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in Ɇ$, $q_{t,i,f} \notin Ɇ$ and $p_{t,i,f} \notin Ȼ$, $q_{t,i,f} \in Ȼ \Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu}\alpha - T_1$ space .

Theorem 4.9: Let $\left(P, \tau_{N_{eu}}\right)$ be any $N_{eu}TS$ and $p_{t,i,f}$, $q_{t,i,f}$ be any two distinct $N_{eu} - pts$ in P . If there exists $N_{eu}gs\alpha^{*} - OS$ $Ɇ$ and $Ȼ$ in $\left(P, \tau_{N_{eu}}\right)$ such that either $p_{t,i,f} \in Ɇ$, $q_{t,i,f} \in Ɇ^{c}$ and $p_{t,i,f} \in Ȼ^{c}$, $q_{t,i,f} \in Ȼ$, then $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_1$ space .

Proof: Let $p_{t,i,f}$, $q_{t,i,f}$ be any two distinct $N_{eu} - pts$ in P . let Ɇ and Ȼ be $N_{eu}gs\alpha^{*} - OS$ in $\left(P, \tau_{N_{eu}}\right)$ such that either $p_{t,i,f} \in$ Ɇ , $q_{t,i,f} \in$ Ɇ^{c} and $p_{t,i,f} \in$ Ȼ^{c}, $q_{t,i,f} \in$ Ȼ $\Rightarrow p_{t,i,f} \in$ Ɇ , $q_{t,i,f} \notin$ Ɇ and $p_{t,i,f} \notin$ Ȼ , $q_{t,i,f} \in$ Ȼ (since Ɇ $\neq$ Ɇ^{c}and Ȼ $\neq$ Ȼ^{c}) $\Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{1}$ space .

Theorem 4.10: A $N_{eu} -$ subspace $\left(Q, \ \sigma_{N_{eu}}\right)$ of a $N_{eu}gs\alpha^{*} - T_{1}$ space $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{1}$ space .

Proof: Let $p_{t,i,f}$, $q_{t,i,f}$ be any two distinct $N_{eu} - pts$ in Q . Then $p_{t,i,f}$, $q_{t,i,f}$ be any two distinct $N_{eu} - pts$ in P . Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{1}$ space , then there exists $N_{eu}gs\alpha^{*} - OS$ Ɇ and Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in$ Ɇ , $q_{t,i,f} \notin$ Ɇ and $p_{t,i,f} \notin$ Ȼ, $q_{t,i,f} \in$ Ȼ . Given $p_{t,i,f} \in Q \cap$ Ɇ $=$ Ɇ , $q_{t,i,f} \notin Q \cap$ Ɇ $=$ Ɇ and $p_{t,i,f} \notin Q \cap$ Ȼ $=$ Ȼ , $q_{t,i,f} \in Q \cap$ Ȼ $=$ Ȼ , where Ɇ and Ȼ are $N_{eu}gs\alpha^{*} - OS$ in $\left(Q, \ \sigma_{N_{eu}}\right)$. Hence , $\left(Q, \ \sigma_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{1}$ space .

Theorem 4.11: Every $N_{eu}gs\alpha^{*} - T_{1}$ space is $N_{eu}gs\alpha^{*} - T_{0}$ space , but not conversely .

Proof: Let $\left(P, \tau_{N_{eu}}\right)$ be any $N_{eu}TS$ in $\left(P, \tau_{N_{eu}}\right)$. Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{1}$ space , then for each pair of distinct $N_{eu} - pts$ $p_{t,i,f}$ and $q_{t,i,f}$ in $P \Rightarrow$ there exists $N_{eu}gs\alpha^{*} - OS$ Ɇ and Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in$ Ɇ , $q_{t,i,f} \notin$ Ɇ and $p_{t,i,f} \notin$ Ȼ , $q_{t,i,f} \in$ Ȼ $\Rightarrow$ there exists $N_{eu}gs\alpha^{*} - OS$ Ɇ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in$ Ɇ and $q_{t,i,f} \notin$ Ɇ $\Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{0}$ space .

Example 4.12: Let $\mathbb{P}$ $=\{p, q\}$ and $\tau_{N_{eu}} = \left\{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ\right\}$, where Ⱥ$=$ $\{\langle p, \ (0.7, \ 0.6, \ 0.5)\rangle, \ \langle q, \ (0.6, \ 0.8, \ 0.4)\rangle\}$ is a $N_{eu}TS$ on $\left(P, \tau_{N_{eu}}\right)$. Let $p_{0.7, 0.6, 0.5}$, $q_{0.7, 0.8, 0.3}$ be any two distinct $N_{eu} - pts$ in P . Then there exist $N_{eu}gs\alpha^{*} - OS$ Ɇ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0.7, 0.6, 0.5} \in$ Ɇ and $q_{0.7, 0.8, 0.3} \notin$ Ɇ , where Ɇ $= \{\langle p, \ (0.7, \ 0.6, \ 0.5)\rangle, \ \langle q, \ (0.6, \ 0.8, \ 0.4)\rangle\} \Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{0}$ space . But $\left(P, \tau_{N_{eu}}\right)$ is not $N_{eu}gs\alpha^{*} - T_{1}$ space , because there exists $N_{eu}gs\alpha^{*} - OS$ Ɇ and Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0.7, 0.6, 0.5} \in$ Ɇ , $q_{0.7, 0.8, 0.3} \notin$ Ɇ and $p_{0.7, 0.6, 0.5} \notin$ Ȼ , $q_{0.7, 0.8, 0.3} \notin$ Ȼ , where Ȼ $=$ $\{\langle p, \ (0.8, \ 0.9, \ 0.3)\rangle, \ \langle q, \ (0.7, \ 0.9, \ 0.3)\rangle\}$.

5. Neutrosophic $gs\alpha^{*} - T_{2}$ Space

Definition 5.1: A $N_{eu}TS\ \left(P, \tau_{N_{eu}}\right)$ is said to be $N_{eu}gs\alpha^{*} - T_{2}$ space (or) $N_{eu}gs\alpha^{*} -$ hausdorff space if for each pair of distinct $N_{eu} - pts\ p_{t,i,f}$ and $q_{t,i,f}$ in P, then there exists $N_{eu}gs\alpha^{*} - OS$ Ɇ and Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f}\in$Ɇ , $q_{t,i,f}\in$Ȼ and Ɇ∩Ȼ $= 0_{N_{eu}}$.

Example 5.2: Let $\mathbb{P} = \{p, q\}$ and $\tau_{N_{eu}} = \left\{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ\right\}$, where Ⱥ$= \{\langle p, (0.4, 0.3, 0.6)\rangle, \langle q, (0.2, 0.4, 0.8)\rangle\}$ is a $N_{eu}TS$ on $\left(P, \tau_{N_{eu}}\right)$. Let $p_{0.4, 0.3, 0.6}$, $q_{0,0,1}$ be any two distinct $N_{eu} - pts$ in P. Then there exist $N_{eu}gs\alpha^{*} - OS$ Ɇ and Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0.4, 0.3, 0.6}\in$Ɇ , $q_{0,0,1}\in$Ȼ and Ɇ∩Ȼ $= 0_{N_{eu}}$, where Ɇ = Ⱥ and Ȼ $= 0_{N_{eu}} \Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{2}$ space.

Theorem 5.3: Every $N_{eu} - T_{2}$ space is $N_{eu}gs\alpha^{*} - T_{2}$ space, but not conversely.

Proof: Let $\left(P, \tau_{N_{eu}}\right)$ be any $N_{eu}TS$ in $\left(P, \tau_{N_{eu}}\right)$. Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu} - T_{2}$ space, then for each pair of distinct $N_{eu} - pts\ p_{t,i,f}$ and $q_{t,i,f}$ in $P \Rightarrow$ there exists $N_{eu} - OS$ Ɇ and Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f}\in$Ɇ , $q_{t,i,f}\in$Ȼ and Ɇ∩Ȼ $= 0_{N_{eu}} \Rightarrow$ there exists $N_{eu}gs\alpha^{*} - OS$ Ɇ and Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f}\in$Ɇ , $q_{t,i,f}\in$Ȼ and Ɇ∩Ȼ $= 0_{N_{eu}} \Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{2}$ space.

Example 5.4: Let $\mathbb{P} = \{p, q\}$ and $\tau_{N_{eu}} = \left\{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ\right\}$, where Ⱥ$= \{\langle p, (0.2, 0.4, 0.6)\rangle, \langle q, (0.4, 0.3, 0.8)\rangle\}$ is a $N_{eu}TS$ on $\left(P, \tau_{N_{eu}}\right)$. Let $p_{0.4, 0.7, 0.5}$, $q_{0,0,1}$ be any two distinct $N_{eu} - pts$ in P. Then there exist $N_{eu}gs\alpha^{*} - OS$ Ɇ and Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0.4, 0.7, 0.5}\in$Ɇ , $q_{0,0,1} \in$ Ȼ and Ɇ∩Ȼ $= 0_{N_{eu}}$, where Ɇ $= \{\langle p, (0.4, 0.7, 0.5)\rangle, \langle q, (0.5, 0.4, 0.7)\rangle\}$ and Ȼ $= 0_{N_{eu}}$ $\Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{2}$ space. But $\left(P, \tau_{N_{eu}}\right)$ is not $N_{eu} - T_{2}$ space, because there exists $N_{eu} - OS$ $Ɇ_{1}$ and $Ȼ_{1}$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0.4, 0.7, 0.5} \notin Ɇ_{1}$, $q_{0,0,1} \in Ȼ_{1}$ and $Ɇ_{1} \cap Ȼ_{1} = 0_{N_{eu}}$, where $Ɇ_{1} =$ Ⱥ and $Ȼ_{1} = 0_{N_{eu}}$.

Theorem 5.5: Let $\left(P, \tau_{N_{eu}}\right)$ be any $N_{eu}TS$. Then $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu} - T_{2}$ space if $\left(P, \tau_{N_{eu}}\right)$ is both $N_{eu}gs\alpha^{*} - T_{2}$ space and $N_{eu}gs\alpha^{*} - T_{\frac{1}{2}}$ space.

Proof: Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{2}$ space, then for each pair of distinct $N_{eu} - pts\ p_{t,i,f}$ and $q_{t,i,f}$ in P $\Rightarrow$ there exists $N_{eu}gs\alpha^{*} - OS$ Ɇ and Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f}\in$Ɇ , $q_{t,i,f} \in$ Ȼ and Ɇ∩Ȼ $= 0_{N_{eu}}$. Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{\frac{1}{2}}$ space, then there exists $N_{eu} - OS$ Ɇ and Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f}\in$Ɇ , $q_{t,i,f} \in$ Ȼ and Ɇ∩Ȼ $= 0_{N_{eu}} \Rightarrow \left(P, \tau_{N_{eu}}\right)$ is $N_{eu} - T_{2}$ space.

Theorem 5.6: Every $N_{eu}\alpha - T_2$ space is $N_{eu}gs\alpha^* - T_2$ space , but not conversely .

Proof: Let $\left(P, \tau_{N_{eu}}\right)$ be any $N_{eu}TS$ in $\left(P, \tau_{N_{eu}}\right)$. Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}\alpha - T_2$ space , then for each pair of distinct $N_{eu} - pts$ $p_{t,i,f}$ and $q_{t,i,f}$ in $P \Rightarrow$ there exists $N_{eu}\alpha - OS$ Ɇ and Ȼ such that $p_{t,i,f} \in$ Ɇ , $q_{t,i,f} \in$ Ȼ and Ɇ∩Ȼ $= 0_{N_{eu}}$ $\Rightarrow$ there exists $N_{eu}gs\alpha^* - OS$ Ɇ and Ȼ such that $p_{t,i,f} \in$ Ɇ , $q_{t,i,f} \in$ Ȼ and Ɇ∩Ȼ $= 0_{N_{eu}}$ $\Rightarrow$ $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^* - T_2$ space .

Example 5.7: Let ℙ $=\{p, q\}$ and $\tau_{N_{eu}} = \left\{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ\right\}$, where Ⱥ$=$ $\{\langle p, (0.4, 0.5, 0.7)\rangle, \langle q, (0.5, 0.4, 0.6)\rangle\}$ is a $N_{eu}TS$ on $\left(P, \tau_{N_{eu}}\right)$. Let $p_{0.4, 0.6, 0.8}$, $q_{0,0,1}$ be any two distinct $N_{eu} - pts$ in P . Then there exist $N_{eu}gs\alpha^* - OS$ Ɇ and Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0.4, 0.6, 0.8} \in$ Ɇ , $q_{0,0,1} \in$ Ȼ and Ɇ∩Ȼ $= 0_{N_{eu}}$, where Ɇ $= \{\langle p, (0.4, 0.6, 0.8)\rangle, \langle q, (0.5, 0.4, 0.6)\rangle\}$ and Ȼ$= 0_{N_{eu}}$ $\Rightarrow$ $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^* - T_2$ space . But $\left(P, \tau_{N_{eu}}\right)$ is not $N_{eu}\alpha - T_2$ space , because there exists $N_{eu}\alpha - OS$ $Ɇ_1$ and $Ȼ_1$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0.4, 0.6, 0.8} \notin Ɇ_1$, $q_{0,0,1} \in Ȼ_1$ and $Ɇ_1 \cap Ȼ_1 = 0_{N_{eu}}$, where $Ɇ_1 =$ Ⱥ and $Ȼ_1 = 0_{N_{eu}}$.

Theorem 5.8: Let $\left(P, \tau_{N_{eu}}\right)$ be any $N_{eu}TS$. Then $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}\alpha - T_2$ space if $\left(P, \tau_{N_{eu}}\right)$ is both $N_{eu}gs\alpha^* - T_2$ space and $N_{eu}gs\alpha^* - T_{\frac{1}{2}}$ space .

Proof: Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^* - T_2$ space , then for each pair of distinct $N_{eu} - pts$ $p_{t,i,f}$ and $q_{t,i,f}$ in P $\Rightarrow$ there exists $N_{eu}gs\alpha^* - OS$ Ɇ and Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in$ Ɇ , $q_{t,i,f} \in$ Ȼ and Ɇ∩Ȼ $= 0_{N_{eu}}$. Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^* - T_{\frac{1}{2}}$ space , then there exists $N_{eu} - OS$ Ɇ and Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in$ Ɇ , $q_{t,i,f} \in$ Ȼ and Ɇ∩Ȼ $= 0_{N_{eu}}$ $\Rightarrow$ then there exists $N_{eu}\alpha - OS$ Ɇ and Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in$ Ɇ , $q_{t,i,f} \in$ Ȼ and Ɇ∩Ȼ $= 0_{N_{eu}}$ $\Rightarrow$ $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}\alpha - T_2$ space .

Theorem 5.9: A N_{eu} $-$ subspace $\left(Q, \sigma_{N_{eu}}\right)$ of a $N_{eu}gs\alpha^* - T_2$ space $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^* - T_2$ space .

Proof: Let $p_{t,i,f}$, $q_{t,i,f}$ be any two distinct $N_{eu} - pts$ in Q . Then $p_{t,i,f}$, $q_{t,i,f}$ be any two distinct $N_{eu} - pts$ in P . Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^* - T_2$ space , then there exists $N_{eu}gs\alpha^* - OS$ Ɇ and Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in$ Ɇ , $q_{t,i,f} \in$ Ȼ and Ɇ∩Ȼ $= 0_{N_{eu}}$. Given $p_{t,i,f} \in Q \cap$ Ɇ $=$ Ɇ , $q_{t,i,f} \in Q \cap$ Ȼ $=$ Ȼ and Ɇ∩Ȼ $= 0_{N_{eu}}$, where Ɇ and Ȼ are $N_{eu}gs\alpha^* - OS$ in $\left(Q, \sigma_{N_{eu}}\right)$. Hence , $\left(Q, \sigma_{N_{eu}}\right)$ is $N_{eu}gs\alpha^* - T_2$ space .

Theorem 5.10: Every $N_{eu}gs\alpha^* - T_2$ space is $N_{eu}gs\alpha^* - T_1$ space , but not conversely .

Proof: Let $\left(P, \tau_{N_{eu}}\right)$ be any $N_{eu}TS$ in $\left(P, \tau_{N_{eu}}\right)$. Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_2$ space , then for each pair of distinct $N_{eu} - pts$ $p_{t,i,f}$ and $q_{t,i,f}$ in P $\Rightarrow$ there exists $N_{eu}gs\alpha^{*} - OS$ Ɇ and Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in$ Ɇ , $q_{t,i,f} \in$ Ȼ and Ɇ∩Ȼ $= 0_{N_{eu}}$ $\Rightarrow$ there exists $N_{eu}gs\alpha^{*} - OS$ Ɇ and Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in$ Ɇ , $q_{t,i,f} \notin$ Ɇ and $p_{t,i,f} \notin$ Ȼ , $q_{t,i,f} \in$ Ȼ $\Rightarrow$ $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_1$ space .

Example 5.11: Let $\mathbb{P}$ $=\{p, q\}$ and $\tau_{N_{eu}} = \left\{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ\right\}$, where Ⱥ= $\{\langle p, (0.2, 0.3, 0.7)\rangle, \langle q, (0.4, 0.5, 0.6)\rangle\}$ is a $N_{eu}TS$ on $\left(P, \tau_{N_{eu}}\right)$. Let $p_{0.2, 0.3, 0.7}$, $q_{0.3, 0.4, 0.7}$ be any two distinct $N_{eu} - pts$ in P . Then there exist $N_{eu}gs\alpha^{*} - OS$ Ɇ and Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0.2, 0.3, 0.7} \in$ Ɇ , $q_{0.3, 0.4, 0.7} \notin$ Ɇ and $p_{0.2, 0.3, 0.7} \notin$ Ȼ , $q_{0.3, 0.4, 0.7} \in$ Ȼ , where Ɇ= Ⱥ and Ȼ $= \{\langle p, (0.3, 0.6, 0.6)\rangle, \langle q, (0.3, 0.4, 0.7)\rangle\}$ $\Rightarrow$ $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_1$ space . But $\left(P, \tau_{N_{eu}}\right)$ is not $N_{eu}gs\alpha^{*} - T_2$ space , because there exists $N_{eu}gs\alpha^{*} - OS$ Ɇ and Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0.2, 0.3, 0.7} \in$ Ɇ , $q_{0.3, 0.4, 0.7} \in$ Ȼ and Ɇ∩Ȼ$\neq 0_{N_{eu}}$.

6. Neutrosophic $gs\alpha^{*} - T_3$ Space

Definition 6.1: A $N_{eu}TS$ $\left(P, \tau_{N_{eu}}\right)$ is said to be $N_{eu}gs\alpha^{*}$ − regular space if for any $N_{eu}gs\alpha^{*} - CS$ Ɇ in $\left(P, \tau_{N_{eu}}\right)$ and for any $N_{eu} - pts$ $p_{t,i,f}$ in P such that $p_{t,i,f} \notin$ Ɇ , then there exists $N_{eu}gs\alpha^{*} - OS$ $Ȼ_1$ and $Ȼ_2$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in Ȼ_1$, Ɇ$\subseteq Ȼ_2$ and $Ȼ_1 \cap Ȼ_2 = 0_{N_{eu}}$.

Definition 6.2: A $N_{eu}TS$ $\left(P, \tau_{N_{eu}}\right)$ is said to be $N_{eu}gs\alpha^{*} - T_3$ space if it is both $N_{eu}gs\alpha^{*}$ − regular space and $N_{eu}gs\alpha^{*} - T_1$ space .

Example 6.3: Let $\mathbb{P}$ $=\{p, q\}$ and $\tau_{N_{eu}} = \left\{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ\right\}$, where Ⱥ= $\{\langle p, (0.5, 0.3, 0.8)\rangle, \langle q, (0.6, 0.4, 0.9)\rangle\}$ is a $N_{eu}TS$ on $\left(P, \tau_{N_{eu}}\right)$. Let $p_{0,0,1}$ be any $N_{eu} - pts$ in P and Ɇ $= \{\langle p, (0.5, 0.4, 0.6)\rangle, \langle q, (0.7, 0.6, 0.4)\rangle\}$ be $N_{eu}gs\alpha^{*} - CS$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0,0,1} \notin$ Ɇ . Then there exist $N_{eu}gs\alpha^{*} - OS$ $Ȼ_1$ and $Ȼ_2$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0,0,1} \in Ȼ_1$, Ɇ$\subseteq Ȼ_2$ and $Ȼ_1 \cap Ȼ_2 = 0_{N_{eu}}$, where $Ȼ_1 = 0_{N_{eu}}$ and $Ȼ_2 = \{\langle p, (0.6, 0.5, 0.4)\rangle, \langle q, (0.8, 0.7, 0.3)\rangle\}$ $\Rightarrow$ $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*}$ − regular space →① . Also , let $q_{0.8, 0.7, 0.3}$ be another $N_{eu} - pts$ in P . Then $p_{0,0,1} \in Ȼ_1$, $q_{0.8, 0.7, 0.3} \notin Ȼ_1$ and $p_{0,0,1} \notin Ȼ_2$, $q_{0.8, 0.7, 0.3} \in Ȼ_2$ $\Rightarrow$ $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_1$ space →② . From ① and ② , $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_3$ space .

Theorem 6.4: Every $N_{eu} - T_3$ space is $N_{eu}gs\alpha^* - T_3$ space , but not conversely .

Proof: Let $\left(P, \tau_{N_{eu}}\right)$ be any $N_{eu}TS$ in $\left(P, \tau_{N_{eu}}\right)$. Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu} - T_3$ space , then it is both N_{eu} − regular space and $N_{eu} - T_1$ space . Given $\left(P, \tau_{N_{eu}}\right)$ is N_{eu} − regular space , then for any $N_{eu} - pts$ $p_{t,i,f}$ in P and any $N_{eu} - CS$ $Ɇ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \notin Ɇ$ ⇒ there exist $N_{eu} - OS$ $₵_1$ and $₵_2$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in ₵_1$, $Ɇ \subseteq ₵_2$ and $₵_1 \cap ₵_2 = 0_{N_{eu}}$ ⇒ for any $N_{eu}gs\alpha^* - CS$ $Ɇ$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \notin Ɇ$, then there exists $N_{eu}gs\alpha^* - OS$ $₵_1$ and $₵_2$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in ₵_1$, $Ɇ \subseteq ₵_2$ and $₵_1 \cap ₵_2 = 0_{N_{eu}}$ ⇒ $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^*$ − regular space →① . Also , Since $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu} - T_1$ space , then $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^* - T_1$ space →② (By theorem 4.3) . From ① and ② , $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^* - T_3$ space .

Example 6.5: Let $\mathbb{P}$ $=\{p, q\}$ and $\tau_{N_{eu}} = \left\{0_{N_{eu}}, 1_{N_{eu}}, Ⱥ\right\}$, where $Ⱥ =$ $\{\langle p, (0.3, 0.2, 0.8)\rangle, \langle q, (0.5, 0.4, 0.6)\rangle\}$ is a $N_{eu}TS$ on $\left(P, \tau_{N_{eu}}\right)$. Let $p_{0,0,1}$ be any $N_{eu} - pts$ in P and $Ɇ = \{\langle p, (0.4, 0.5, 0.3)\rangle, \langle q, (0.4, 0.3, 0.2)\rangle\}$ be $N_{eu}gs\alpha^* - CS$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0,0,1} \notin Ɇ$. Then there exist $N_{eu}gs\alpha^* - OS$ $₵_1$ and $₵_2$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0,0,1} \in ₵_1$, $Ɇ \subseteq ₵_2$ and $₵_1 \cap ₵_2 = 0_{N_{eu}}$, where $₵_1 = 0_{N_{eu}}$ and $₵_2 = \{\langle p, (0.5, 0.6, 0.2)\rangle, \langle q, (0.6, 0.4, 0.1)\rangle\}$ ⇒ $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^*$ − regular space → ① . Also , let $q_{0.6,0.4,0.1}$ be another $N_{eu} - pts$ in P . Then there exists $p_{0,0,1} \in ₵_1$, $q_{0.6,0.4,0.1} \notin ₵_1$ and $p_{0,0,1} \notin ₵_2$, $q_{0.6,0.4,0.1} \in ₵_2$ ⇒ $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^* - T_1$ space → ② . From ① and ② , $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^* - T_3$ space . But $\left(P, \tau_{N_{eu}}\right)$ is not $N_{eu} - T_3$ space . Let $₵ =$ $\{\langle p, (0.2, 0.1, 0.9)\rangle, \langle q, (0.4, 0.3, 0.7)\rangle\}$ be $N_{eu} - CS$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0,0,1} \notin ₵$. Then there exist $N_{eu} - OS$ $Ɇ_1$ and $Ɇ_2$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{0,0,1} \in Ɇ_1$, $₵ \subseteq Ɇ_2$ and $Ɇ_1 \cap Ɇ_2 = 0_{N_{eu}}$, where $Ɇ_1 = 0_{N_{eu}}$ and $Ɇ_2$ $= \{\langle p, (0.3, 0.2, 0.8)\rangle, \langle q, (0.5, 0.4, 0.6)\rangle\}$ ⇒ $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^*$ − regular space →③ . Also , there exist $p_{0,0,1} \in Ɇ_1$, $q_{0.6,0.4,0.1} \notin Ɇ_1$ and $p_{0,0,1} \notin Ɇ_2$, $q_{0.6,0.4,0.1} \notin Ɇ_2$ ⇒ $\left(P, \tau_{N_{eu}}\right)$ is not $N_{eu}gs\alpha^* - T_1$ space →④ . From ③ and ④ , $\left(P, \tau_{N_{eu}}\right)$ is not $N_{eu} - T_3$ space .

Theorem 6.6: Let $\left(P, \tau_{N_{eu}}\right)$ be any $N_{eu}TS$. Then $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu} - T_3$ space if $\left(P, \tau_{N_{eu}}\right)$ is both $N_{eu}gs\alpha^* - T_3$ space and $N_{eu}gs\alpha^* - T_{\frac{1}{2}}$ space .

Proof: Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{3}$ space , then for any $N_{eu} - pts\ p_{t,i,f}$ in P and any $N_{eu}gs\alpha^{*} - CS$ Ɇ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f}$∉Ɇ . Now , there exists $N_{eu}gs\alpha^{*} - OS$ Ȼ$_1$ and Ȼ$_2$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in$ Ȼ$_1$, Ɇ⊆Ȼ$_2$ and Ȼ$_1$ ∩ Ȼ$_2$ $= 0_{N_{eu}}$. Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{\frac{1}{2}}$ space , then for any $N_{eu} - CS$ Ɇ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f}$∉Ɇ ⇒ there exists $N_{eu} - OS$ Ȼ$_1$ and Ȼ$_2$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in$ Ȼ$_1$, Ɇ⊆Ȼ$_2$ and Ȼ$_1$ ∩ Ȼ$_2$ $= 0_{N_{eu}}$ ⇒ $\left(P, \tau_{N_{eu}}\right)$ is N_{eu} − regular space →① . Also , since $\left(P, \tau_{N_{eu}}\right)$ is both $N_{eu}gs\alpha^{*} - T_{1}$ space and $N_{eu}gs\alpha^{*} - T_{\frac{1}{2}}$ space , then $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu} - T_{1}$ space →② (By theorem 4.5). From ① and ② , $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu} - T_{3}$ space .

Theorem 6.7: A N_{eu} − subspace $\left(Q,\ \sigma_{N_{eu}}\right)$ of a $N_{eu}gs\alpha^{*} - T_{3}$ space $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{3}$ space .

Proof: By theorem 4.10 , A N_{eu} − subspace $\left(Q,\ \sigma_{N_{eu}}\right)$ of a $N_{eu}gs\alpha^{*} - T_{1}$ space $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{1}$ space →① . Let $p_{t,i,f}$ be any $N_{eu} - pts$ in Q and Ɇ be any $N_{eu}gs\alpha^{*} - CS$ in $\left(Q,\ \sigma_{N_{eu}}\right)$ such that $p_{t,i,f}$ ∉ Ɇ . Then Ɇ= Q∩Ȼ for some $N_{eu}gs\alpha^{*} - CS$ Ȼ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f}$∉Q∩Ȼ = Ȼ . Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{3}$ space , then there exists $N_{eu}gs\alpha^{*} - OS$ Ȼ$_1$ and Ȼ$_2$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in$ Ȼ$_1$, Ȼ⊆Ȼ$_2$and Ȼ$_1$ ∩ Ȼ$_2$ $= 0_{N_{eu}}$. Take Ɇ$_1$ = Q∩Ȼ$_1$ and Ɇ$_2$ = Q∩Ȼ$_2$, then Ɇ$_1$ and Ɇ$_2$ are $N_{eu}gs\alpha^{*} - OS$ in $\left(Q,\ \sigma_{N_{eu}}\right)$ such that $p_{t,i,f} \in$ Ɇ$_1$, Ɇ⊆Ɇ$_2$ and Ɇ$_1$ ∩ Ɇ$_2$ ⊆ Ȼ$_1$ ∩ Ȼ$_2$ $= 0_{N_{eu}}$ ⇒ $\left(Q,\ \sigma_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*}$ − regular space →② . From ① and ② , $\left(Q,\ \sigma_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{3}$ space .

Theorem 6.8: Every $N_{eu}gs\alpha^{*} - T_{3}$ space is $N_{eu}gs\alpha^{*} - T_{2}$ space .

Proof: Let $\left(P, \tau_{N_{eu}}\right)$ be any $N_{eu}TS$ in $\left(P, \tau_{N_{eu}}\right)$. Given $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{3}$ space , then for each pair of distinct $N_{eu} - pts\ p_{t,i,f}$ and $q_{t,i,f}$ in P and for any $N_{eu}gs\alpha^{*} - CS$ Ɇ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f}$∉Ɇ ⇒ there exists $N_{eu}gs\alpha^{*} - OS$ Ȼ$_1$ and Ȼ$_2$ in $\left(P, \tau_{N_{eu}}\right)$ such that Ɇ⊆Ȼ$_2$, Ȼ$_1$ ∩ Ȼ$_2$ $= 0_{N_{eu}}$, $p_{t,i,f} \in$ Ȼ$_1$, $q_{t,i,f} \notin$ Ȼ$_1$ and $p_{t,i,f} \notin$ Ȼ$_2$, $q_{t,i,f} \in$ Ȼ$_2$ ⇒ there exists $N_{eu}gs\alpha^{*} - OS$ Ȼ$_1$ and Ȼ$_2$ in $\left(P, \tau_{N_{eu}}\right)$ such that $p_{t,i,f} \in$ Ȼ$_1$, $q_{t,i,f} \in$ Ȼ$_2$ and Ȼ$_1$ ∩ Ȼ$_2$ $= 0_{N_{eu}}$ ⇒ $\left(P, \tau_{N_{eu}}\right)$ is $N_{eu}gs\alpha^{*} - T_{2}$ space .

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